

A Cubical Path from Algebra to Analysis

Formal Foundry Research Group

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Abstract

We connect algebraic interfaces to constructive analysis in Cubical Agda, tracing a path through ordered rings, premetric spaces, completions, continuity, and real-number infrastructure. The work demonstrates how informal mathematical intent can be decomposed into reusable checked components that support later high-stakes formalization work. It also shows how reusable real-number components can carry definitions across a larger development.

“A Cubical Path from Algebra to Analysis” is the strongest concrete research-paper item for the site.

The work develops constructive analysis in Cubical Agda by connecting algebraic foundations to analytic structure. Its path moves through pseudolattices, ordered commutative rings, Archimedean rings, premetric spaces, higher inductive-inductive completions, Lipschitz maps, continuity, and HoTT Book Cauchy reals.

Why It Matters

The page should show that Formal Foundry’s proof-assistant work is not limited to product demos. Real analysis forces definitions, algebraic interfaces, order structures, limits, completions, and theorem dependencies to line up across a large development.

That makes it a useful credibility artifact for high-stakes formalization work. It demonstrates the kind of discipline required when informal mathematical intent has to become checked code.

Development Themes

The broader real-number development includes work around:

- ordered commutative rings and Archimedean rings;
- premetric spaces and completion;
- rationals, dyadics, and Cauchy reals;
- order, apartness, reciprocals, and completeness;
- pointwise and uniform continuity;
- derivatives, limits, chain rule, Rolle’s theorem, and mean value theorem;
- Riemann integration with tagged partitions and refinement;
- exponential, trigonometric, roots, and constant-related infrastructure.

Best Page Assets

- a diagram of the path from algebra to analysis;

- a small module map from the real-number checkout;
- selected theorem statements checked against the original paper and Agda modules;
- generated Agda HTML served locally or exported to the public artifact host;
- the TYPES 2026 slides as a reader-friendly companion.